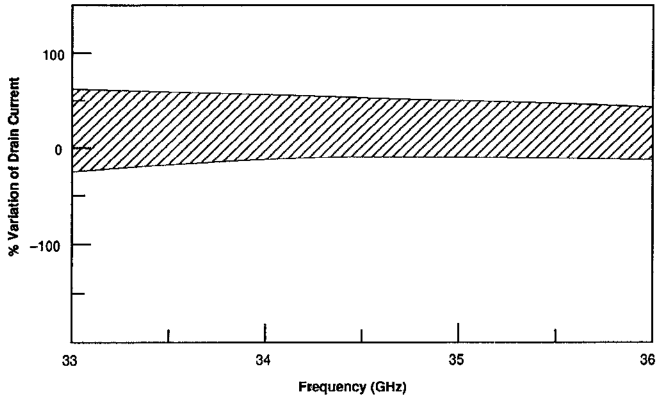

Fig. 7. Bias sensitivity of the amplifier at VDS = 5V. The drain current is varied to change the gain by ±0.5 dB (max) from the maximum efficiency point. The drain current is changed with gate voltage. The maximum efficiency occurs with $P_{in} \sim 10$ dBm; so the input is maintained at 10 dBm. The amplifier can be maintained well under saturation with more than 50% variation in drain current over the frequency range 33-36 GHz.

with the highest efficiency. The response is shown in Fig. 6(a) for five amplifiers with the same drain and gate voltages (VDS = 5V VGS = −1.35V) over a narrower band 33-36 GHz, the nominal input is 10-11 dBm (exactly 10 dBm at 34.5 GHz). The five amplifiers are chosen from five reticles at random approximately from the center of the wafer. The response shows the repeatability of the design at least from the same wafer over the band of interest. Fig. 6(b) and (c) show the power saturation characteristics at 33 GHz and 35 GHz, respectively. The efficiency reaches as high as 28% at 33 GHz and 21% at 35 GHz at a high compression level (3-4 dB). At those efficiencies, the power outputs are 137 mW at 33 GHz and 113 mW at 35 GHz, and the drain currents are 91 mA at 33 GHz and 95 mA at 35 GHz and corresponding P_{in}'s are 10 dBm at 33 GHz and 11 dBm at 35 GHz. Finally Fig. 7 shows the bias sensitivity of the amplifiers. The drain bias is fixed at VDS = 5V and gate bias varied to make the gain change by ±0.5 dB at the maximum efficiency bias point. At higher drain current, the gain first increases before it starts dropping; the efficiency also degrades with the gain variation. All the results reported here are taken with RF probes with the chip placed on a metal block without being soldered. The yield window, being application specific, was set at $G \pm 2$ dB, where G is the gain with nominal 10 dBm input. G in our case was set at 10 dB over 33-35 GHz. The results reported here are from a nominal wafer and also the best wafer that we measured.

IV. Conclusion

A broad *Ka*-band three-stage power MMIC is successfully demonstrated which is designed with MBE MESFET devices with moderate doping levels and device transconductance. The design is quite tolerant to process and bias variations. The performance is suitable for the applications requiring high volume, low cost and manufacturable process. The amplifier shows fairly high efficiency over a broadband, which is more than adequate in smart ammunition applications.

Acknowledgment

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Criteria for the Onset of Oscillation In Microwave Circuits

Robert W. Jackson

***Abstract*—A commonly used criterion for oscillator startup is demonstrated not to be universally valid. In order to investigate startup conditions, the Nyquist stability criterion is written in terms of microwave quantities. It is shown that widely available microwave CAD can be used to create Nyquist stability plots. Since the Nyquist criterion gives only global stability information, a convenient graphical criterion is developed to determine whether an oscillation will start up near a particular resonance frequency.**

I. Introduction

Many texts and papers use the following criteria for oscillation start up at resonance,

$$|\Gamma_d||\Gamma_c| > 1 \quad \text{and} \quad \phi_d + \phi_c = 0 \tag{1}$$

where Γ_d is the reflection coefficient seen looking into an active device, Γ_c is the reflection coefficient seen looking into a passive resonating circuit, and ϕ_d, ϕ_c are the angles associated with each reflection coefficient. The resonance frequency is defined to be the frequency at which ϕ_d and ϕ_c sum to zero.

This condition is intuitively attractive, but it does not accurately predict instability or stability in all cases. For example, consider the two circuits in Fig. 1. These simple circuits are easily analyzed by conventional circuit theory techniques. Circuit A has left half plane poles and thus is stable. Circuit B has right half plane poles and is unstable. The reflection coefficients Γ_d and Γ_c are 6 and 0.091, respectively at resonance (50 Ω reference). Criteria (1) predict that both circuits will be stable. This is clearly the wrong conclusion for circuit B.

In this paper, we write the well-known Nyquist stability criterion in terms of quantities which are commonly used in microwave active circuit design. In Section I, it will be shown that the Nyquist plot for microwave circuits can be easily generated using commercially available microwave CAD.

Although the Nyquist criterion rigorously determines the stability of a circuit, it does not give any information about oscillation frequencies. It just answers the question, "Is it stable or not?" In Section II, we consider the case where a circuit is assumed to be

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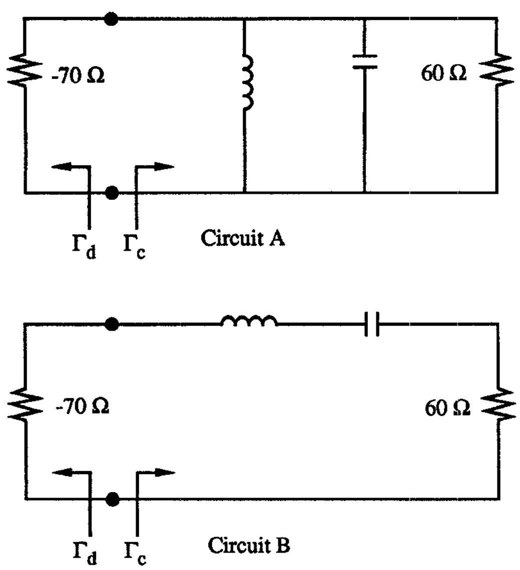


Fig. 1. Two simple, ideal circuits. Equation (1) incorrectly predicts that circuit B is stable.

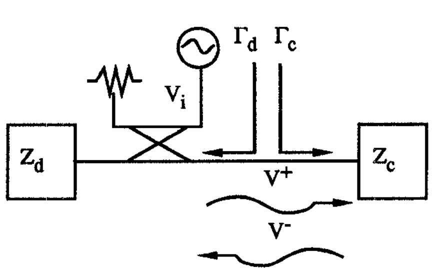


Fig. 2. A generic, driven microwave system.

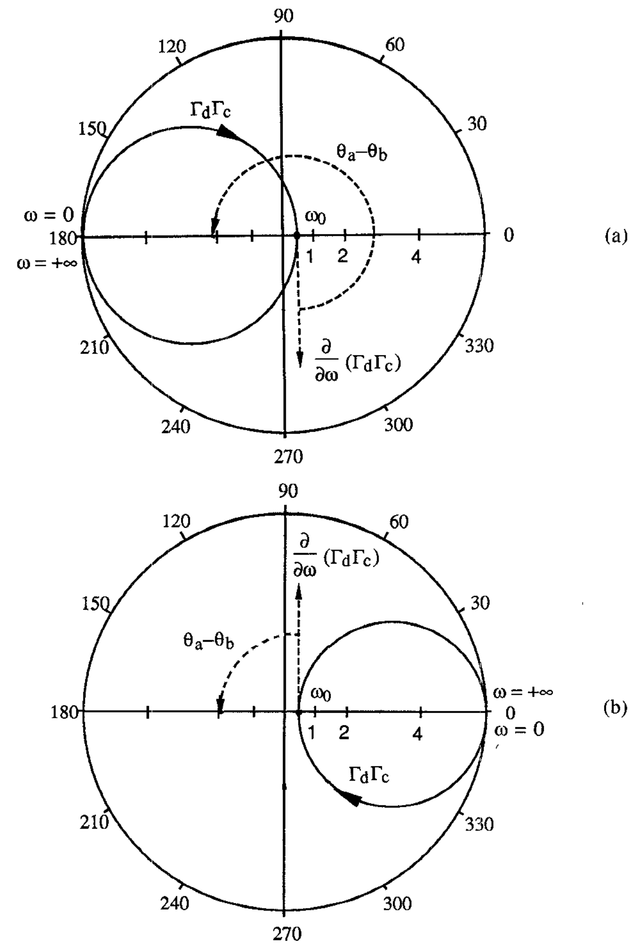


Fig. 3. (a) Nyquist plot (solid line) for circuit A in Fig. 1 indicating a stable circuit. (b) Nyquist plot for circuit B circling +1 thus indicating instability. The angles (dashed lines) refer to the graphical technique described in Section III.

slightly unstable and establish graphical criteria for determining stability or instability at a resonance frequency.

II. The Nyquist Stability Criterion for Microwave Circuits

The transfer function of a generic feedback system can be written in the form

$$X_0 = \frac{G(s)}{1 + G(s)H(s)} X_i \tag{2}$$

where the X values are generic input or output amplitudes, $G(s)$ is the gain function, and $H(s)$ is the feedback function. The Nyquist stability criterion for such a system is as follows [1]. For a stable open-loop transfer function, $GH(s)$, the closed-loop system will be unstable if the point -1 is encircled at least once by the polar plot of $GH(j\omega)$ for $-\infty < \omega < \infty$.

Fig. 2 shows a microwave system which can be described in a manner analogous to (2). V_i is an input wave launched by an idealized wave source consisting of a directional coupler and source (The idealized wave source assumes that the coupler coupling factor approaches zero and the source amplitude approaches infinity). V^+ can be considered the output. Thus the system function is determined by

$$V^+ = \Gamma_d(V_i + V^-) = \Gamma_d V_i + \Gamma_d \Gamma_c V^+$$

$$V^+ = \frac{\Gamma_d(s)}{1 - \Gamma_d(s)\Gamma_c(s)} V_i. \tag{3}$$

The system is unstable if V^+ grows from some initial value when V_i is zero. Assuming Γ_d represents a *potentially* unstable device (unstable only *after* connecting the resonator), the overall system will be unstable if the point $+1$ is encircled at least once by the polar plot of $\Gamma_d\Gamma_c(j\omega)$ for $-\infty < \omega < \infty$. This indicates that the overall system has right half plane poles. Fig. 3(a) and (b) shows the Nyquist plot for the examples in Fig. 1(a) and (b). The Nyquist plot correctly predicts the stability of circuit A and the instability of circuit B. (Note that the plot for $0 < \omega < \infty$ traverses the same path as for $-\infty < \omega < 0$.)

The evaluation of the product $\Gamma_c\Gamma_d$ is easily accomplished using the standard features of microwave CAD packages. Only an ideal circulator model is necessary. If such a model is not available, most packages have a user-defined three-port which can represent an ideal circulator by defining $S_{12} = S_{23} = S_{31} = 1$ with all other entries set equal to zero. Fig. 4 shows the circuit block diagram. Γ_d and Γ_c could be the result of intricate circuit models or they could result from measured data. The software is used to plot the reflection coefficient seen at circulator port 1 versus frequency on a polar plot. This results in half of the Nyquist plot ($0 < \omega < +\infty$). The other half is just the mirror image of the first reflected around the x axis, since $\Gamma_d\Gamma_c(j\omega) = \Gamma_d^*\Gamma_c^*(-j\omega)$. The entire curve is easy to envision and sketch in on a hard copy output of the first half plot.

As a further example, consider the reactively matched amplifier

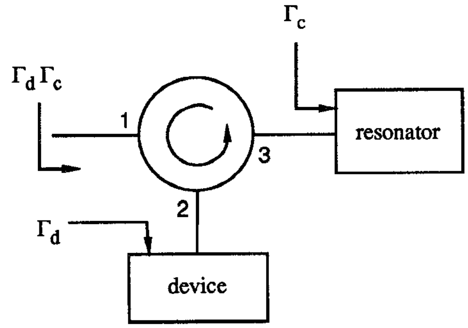
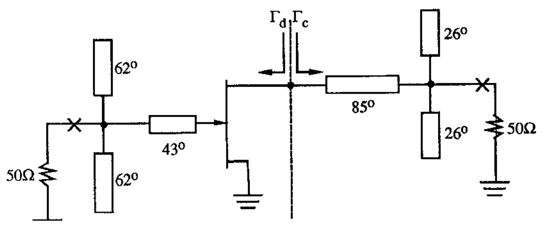


Fig. 4. CAD circuit for evaluation of $\Gamma_d\Gamma_c$ product versus frequency.



F	S11	S21	S12	S22
2ghz	.98 -29	3.54 157	.031 73	.80 -11
6ghz	.88 -80	2.90 119	.074 46	.72 -29
10ghz	.79 -117	2.27 90	.088 29	.65 -44
14ghz	.75 -146	1.83 66	.086 20	.60 -58
18ghz	.74 -169	1.53 46	.080 20	.57 -74

Fig. 5. An example of a two port FET circuit including the FET scattering parameters. All transmission line impedances are 50 Ω and f_o is 10 GHz.

in Fig. 5. The accompanying table shows the device S parameters over a typical range. The stability factor for this device is less than 1 below 17 GHz. Thus the device, and also the two port created by adding the reactive matching, is *potentially* unstable. If terminations for the two port are specified, the Nyquist criterion can then be applied to determine whether the resulting circuit is *actually* unstable. In this example circuit the reference plane shown is chosen arbitrarily and Γ_d, Γ_c are defined with respect to it. Both reflection coefficients are defined, again arbitrarily, with respect to a 50 Ω reference impedance. In case A, assume the two port is terminated as shown with 50 Ω terminations. For these terminations, the plot in Fig. 6 shows that $\Gamma_d\Gamma_c(j\omega)$ does not encircle $+1$ and thus the circuit is stable. Note that only the parts of the curve between 1 GHz and 18 GHz and between -1 GHz and -18 GHz (negative range not plotted) can be determined from the given S parameters. Above 17 GHz the device is unconditionally stable and thus the product $|\Gamma_d\Gamma_c|$ is always less than one and cannot circle $+1$ if it has not already. Below 1 GHz some assumption must be made about the behavior of the reflection coefficients. In this example, it is assumed that $|\Gamma_d\Gamma_c|$ remains less than one down to dc. This low frequency assumption is risky as bias circuitry can often result in instability due to encirclements of $+1$ at low frequencies. For case B we replace the 50 Ω termination at the second port with a 45° length of 50 Ω transmission line terminated in an open circuit. Curve B in Fig. 6 shows that the Nyquist plot now encircles $+1$ and thus the circuit is unstable. As an aside it should be noted that this circuit is designed as an amplifier with a good input match and a gain greater than the maximum stable gain. The reflection coefficient magnitude seen looking into port 2 is greater than 1, but for a variety of terminations, including 50 Ω, the circuit is stable. In practice such a design is not usually considered since it is very sensitive to device parameters, is narrow band, and would not be amenable to cascading.

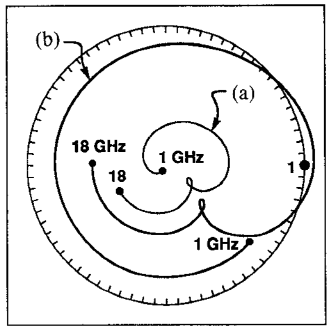


Fig. 6. Polar plot of $\Gamma_d\Gamma_c$ versus frequency for the two port circuit in Fig. 5. (a) With 50 Ω terminations. (b) When the 50 Ω termination on port 2 is replaced with a 50 Ω, 45° open circuit stub.

III. Graphical Stability Criterion for Slightly Unstable Circuits

Unfortunately the Nyquist method does not give any information about oscillation frequency. This is important if one is designing a circuit to oscillate at a particular frequency and it is necessary to determine if the oscillation will start up in response to ambient noise. Criterion (1) is often used for this purpose, but as we have seen, it does not always make correct predictions. We desire a graphical approach such as Kurokawa's (nonlinear operating point stability) [2], [3] that can test whether a linear circuit has an unstable pole located near a particular real frequency. In what follows, a graphical criterion is derived that is valid for circuits having right half plane poles located near the $j\omega$ axis.

The characteristic equation for the poles of the system function in (3) can be written:

$$1 - \Gamma_d(s)\Gamma_c(s) = 0. \tag{4}$$

If one had analytical expressions for both quantities in this equation, a root finder could be used to locate the complex zeros (poles of (3)). The stability and startup frequency could thereby be established by the zero location. However, Γ_d and Γ_c are usually known only for values of s along the imaginary axis and are not known elsewhere in the complex plane. This is either because they result from experimental data or because CAD programs have not been set up to evaluate impedance as a function of complex frequencies.

In what follows it is assumed that when a solution to equation (4) is located near the $j\omega$ axis it will be located near the resonance frequency, ω_0, as defined by

$$\phi_d(j\omega_0) + \phi_c(j\omega_0) = 0. \tag{5a}$$

With this in mind it is useful to write the solution to (4) in the form

$$s = j\omega_0 + j\Delta\omega + \sigma \tag{5b}$$

where, under the stated assumption, $\Delta\omega$ and σ are small relative to ω_0. Equation (4) can then be expanded around ω_0 such that,

$$1 - \Gamma_d(j\omega_0)\Gamma_c(j\omega_0) - \frac{\partial}{\partial\omega}(\Gamma_d\Gamma_c)\bigg|_{\omega\approx\omega_0}(\Delta\omega - j\sigma) \approx 0. \tag{6}$$

Setting the real and imaginary parts of this equation to zero results in two equations for the unknowns $\Delta\omega$ and σ. Using one of these equations to eliminate $\Delta\omega$ results in the following equation for σ,

$$\sigma\left|\frac{\partial}{\partial\omega_0}(\Gamma_d\Gamma_c)\right|^2 = (1 - \Gamma_d\Gamma_c)\,\mathrm{Im}\left[\frac{\partial}{\partial\omega}(\Gamma_d\Gamma_c)\right] \tag{7a}$$

$$= \mathrm{Im}\left[(\Gamma_d\Gamma_c - 1)\frac{\partial}{\partial\omega}(\Gamma_d\Gamma_c)^*\right] \tag{7b}$$

$$= |\Gamma_d\Gamma_c - 1| \left|\frac{\partial}{\partial \omega}(\Gamma_d\Gamma_c)\right| \sin(\theta_a - \theta_b) \quad (7c)$$

where $\theta_a \equiv \angle(\Gamma_d\Gamma_c - 1)$ and $\theta_b = \angle \partial/\partial\omega_0(\Gamma_d\Gamma_c)$. Using (7), σ was calculated and compared to the exact values for circuits A and B in Fig. 1. Errors of 6% and 8% were obtained independent of the values used for C and L. The main point of interest in this section is the sign of σ. A positive σ means that a pole of (3) is in the right half plane and therefore the circuit is unstable.

A convenient graphical criterion for determining the sign of σ can be found by manipulating by the right hand side of (7) where, as noted previously, all quantities are evaluated at ω_0 as defined by (5a). Note that the condition $|\Gamma_d||\Gamma_c| > 1$ indicates instability only if the imaginary part of the frequency derivative in (7a) is positive. This last condition is violated in circuit B. The expression in (7c) indicates a graphical method of determining stability. Referring to Fig. 3, first form a vector which is tangent to the $\Gamma_d\Gamma_c$ curve at ω_0 and pointing in the direction of increasing ω. The angle $\theta_a - \theta_b$ is swept by turning this vector in a counterclockwise direction around its origin to the direction of the vector that points from 1 to $\Gamma_d\Gamma_c$ at ω_0. The angle is indicated in Fig. 3. Equation (7c) therefore shows that the only requirement for instability is that $180° > \theta_a - \theta_b > 0$. This would replace the not always valid equation (1).

A more convenient graphical approach can be determined for the typical case where a circuit can be split in such a way that Γ_d is approximately constant relative to the frequency dependence of Γ_c. By manipulating the expression in (7b), the following expression for instability can be written:

$$\mathrm{Im}\left[\left(\Gamma_c - \frac{1}{\Gamma_d}\right)\left(\frac{\partial}{\partial \omega}\Gamma_c\right)^*\right] > 0. \quad (8)$$

If $\phi_a \equiv \angle(\Gamma_c - 1/\Gamma_d)$ and $\phi_b \equiv \angle(\partial\Gamma_c/\partial\omega_0)$, criterion (8) can be simplified to

$$\sin(\phi_a - \phi_b) > 0. \quad (9)$$

Referring to Fig. 7, form a vector which points from the point $1/\Gamma_d$ to the point $\Gamma_c(j\omega_0)$. The direction of this vector defines ϕ_a. Draw another vector tangent to the Γ_c versus ω curve at ω_0 pointing in the direction of increasing ω_0. The direction of this vector defines ϕ_b. The angle $\phi_a - \phi_b$ is swept by turning the latter vector counterclockwise around the point $\Gamma_c(j\omega_0)$ until it is pointing in the direction of the former vector. If $180° > \phi_a - \phi_b > 0$, the circuit has a right half plane pole near ω_0 and is therefore unstable. This graphical method is usually more convenient than the one in the previous paragraph since it deals with quantities which remain inside the $|\Gamma| = 1$ circle. However, it does assume that Γ_d is approximately frequency independent.

IV. Conclusion

In this paper we have shown that a commonly used condition for instability is not universally valid. The Nyquist stability criterion has been written in a form that is convenient for microwave usage. A technique for producing Nyquist plots on commercially available microwave CAD packages has been described. An approximate expression was derived which locates a complex pole existing near a circuit resonance. Lastly, a graphical test was described for determining whether a circuit will start oscillating near a particular resonance frequency.

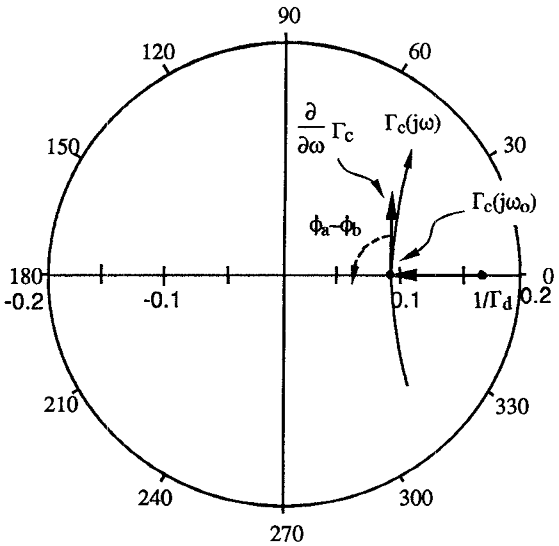


Fig. 7. Illustration of a graphical evaluation of stability near a resonance. The plot corresponds to the unstable circuit B in Fig. 1.

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A General Approach for the *S*-Parameter Design of Oscillators with 1 and 2-Port Active Devices

R. D. Martinez and R. C. Compton

***Abstract*—This paper introduces a circular function that serves as a basis for deciding if a circuit will continuously oscillate. The circular function is derived from the signal flow graph of the circuit including the external load. Any node in the flow-graph can be split into two nodes, one of which contains incoming branches and the other containing outgoing branches. The circular function is then the transfer function between the two nodes, and it can be measured or simulated by looking at the reflection coefficient of a circulator inserted at the node that was originally split. Oscillations occur when the circular function is unity. Stability of these oscillations is determined by con-**

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